Sydney Girls High School
2022
Trial Higher School Certificate
Examination

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using a black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- In Questions 11-14, show relevant mathematical reasoning and/or calculations

Total marks: 70

Section I – 10 marks (pages 3-6)

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 8-15)

- Attempt Questions 11-14
- Allow about 1 hour and 45 minutes for this section

Name: …………………………………………………… Teacher: ……………………………………………………	**THIS IS A TRIAL PAPER ONLY** It does not necessarily reflect the format or the content of the 2022 HSC Examination Paper in this subject.

Section 1

10 marks

Attempt Questions 1-10

Use the Multiple-choice answer sheet for questions 1-10

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1) Given $f(x) = \sqrt{x} - 3$, what are the domain and range of $y = f^{-1}(x)$?

A. $x \geq -3, \quad y \geq 0$

B. $x \geq -3, \quad y \geq -3$

C. $x \geq 0, \quad y \geq 0$

D. $x \geq 0, \quad y \geq -3$

2) What is the derivative of $f(x) = \tan^{-1}\left(\frac{1}{x}\right)$?

A. $\frac{-1}{1+x^2}$

B. $\frac{-x^2}{1+x^2}$

C. $\frac{1}{1+x^2}$

D. $\frac{x^2}{1+x^2}$

3) Terry is one of 5 candidates in an election. If there are 106 voters what is the minimum number of votes she could receive and still win?

A. 20

B. 21

C. 22

D. 23

4) What is the solution set of the inequality $x(x^2 - 9) < x$?

A. $(-\sqrt{8}, 0) \cup (\sqrt{8}, \infty)$

B. $(-\infty, -\sqrt{10}) \cup (0, \sqrt{10})$

C. $(-\infty, -\sqrt{8}) \cup (0, \sqrt{8})$

D. $(-\sqrt{10}, 0) \cup (\sqrt{10}, \infty)$

5) A curve in the Cartesian plane is defined parametrically by the equations $x = t^3 + t$ and $y = t^4 + 2t^2$.

The equation of the tangent to the curve at $t = 1$ is:

A. $y = 2x$

B. $y = 8x$

C. $y = 2x - 1$

D. $y = 8x - 13$

6) What is the component of the force $\underset{\sim}{F} = a\,\underset{\sim}{i} + b\,\underset{\sim}{j}$ where a and b are non-zero real constants, in the direction of the vector $\underset{\sim}{w} = \underset{\sim}{i} + \underset{\sim}{j}$?

A. $\left(\frac{a+b}{2}\right)\underset{\sim}{w}$

B. $\left(\frac{1}{a+b}\right)\underset{\sim}{F}$

C. $\left(\frac{a+b}{a^2+b^2}\right)\underset{\sim}{F}$

D. $\left(\frac{a+b}{\sqrt{2}}\right)\underset{\sim}{w}$

7) Let R be the region between the graphs of $y = 1$ and $y = \sin x$ from $x = 0$ to $x = \frac{\pi}{2}$.
The volume of the solid obtained by revolving R about the x −axis is given by:

A. $$2\pi \int_0^{\frac{\pi}{2}} x \sin x \; dx$$

B. $$\pi \int_0^{\frac{\pi}{2}} (1 - \sin x)^2 \, dx$$

C. $$\pi \int_0^{\frac{\pi}{2}} \sin^2 x \; dx$$

D. $$\pi \int_0^{\frac{\pi}{2}} (1 - \sin^2 x) \, dx$$

8) In how many different ways can three girls and three boys be seated on a bench for a photograph if the girls must all sit apart?

A. 72

B. 120

C. 144

D. 576

9) Given $\tan A - \tan B = \cot A - \cot B - 1 = m$, which of the following is the correct expression for $\tan(A - B)$ in terms of m.

A. $\frac{m(m+1)}{2m+1}$

B. $m(m+1)$

C. $-\frac{m(m+1)}{2m+1}$

D. $-m(m+1)$

10) Given that x and y are both prime numbers and integers larger than 1, how many terms of the binomial expansion $\left(\sqrt{x} + \sqrt[5]{y}\right)^{301}$ will be rational?

A. 151

B. 60

C. 31

D. 30

Section II

60 marks

Attempt questions 11-14

Start each question on a NEW piece of paper

Question 11 (15 marks)

Use a new sheet of paper.

(a) Let $\overrightarrow{OA} = 3\underset{\sim}{i} - 5\underset{\sim}{j}$ and $\overrightarrow{OB} = 2\underset{\sim}{i} + 6\underset{\sim}{j}$.

(i) Find $|\overrightarrow{OA}|$. 1

(ii) Find $\overrightarrow{AB}$. 1

(iii) Find the size of $\angle AOB$, correct to the nearest degree. 2

(b) Find the value of the following integral, giving your answer in terms of π. 2

$$\int_0^{\frac{1}{3}} \frac{1}{\sqrt{4-9x^2}}\,dx$$

(c) A polynomial is given by $P(x) = x^3 + ax^2 - 6ax + b$. 3

Find the values of a and b given the following conditions.

- $x = 2$ is a root of the polynomial

  <u>and</u>

- the remainder is 2 when $P(x)$ is divided by $(x - 1)$

Question 11 continues on the following page

Question 11 (continued)

(d) (i) Given $t = \tan\left(\frac{\theta}{2}\right)$, show that: 1

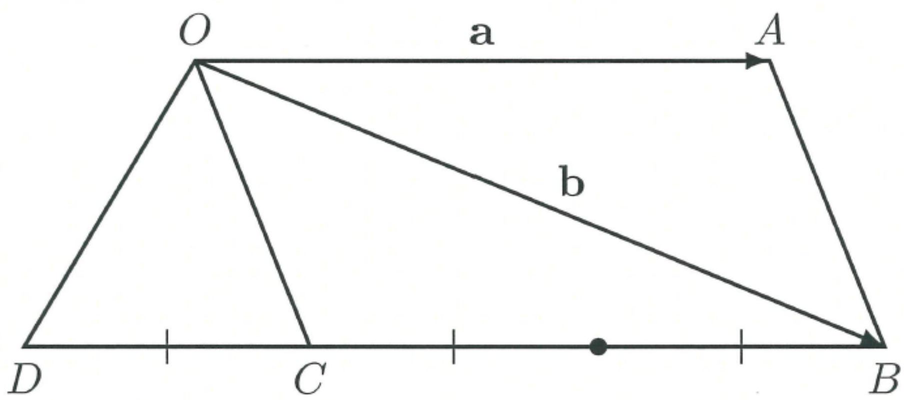
$$\sin\left(\frac{\theta}{2}\right) = \frac{t}{\sqrt{1+t^2}} \; .$$

(ii) Hence solve the following equation for θ, where $\theta \in [0, 2\pi]$. 3

$$\cos\theta - \sin\left(\frac{\theta}{2}\right) = 0$$

Express your solutions in terms of π.

(e) In the diagram below, $OABC$ is a parallelogram and D, C and B are collinear such that $DC:CB = 1:2$. Given $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OB} = \underset{\sim}{b}$, find an expression for $\overrightarrow{AD}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$. 2



Question 12 (15 marks)

Use a new sheet of paper

(a) The rate at which a body warms in air is proportional to the difference between its temperature T and the constant temperature A of the surrounding air.
This rate can be expressed using the differential equation:

$$\frac{dT}{dt} = k(T - A)$$

where t is the time in minutes and k is a constant.

(i) Show that $T = A + Pe^{kt}$, where P is a constant, is a solution of the differential equation. 1

(ii) At the time a cheesecake is removed from a fridge, it is $2°C$. 3
After 15 minutes, it has warmed to $10°C$.
Given the air temperature around the cheesecake is $25°C$, find the temperature of the cheesecake after a further 15 minutes.
Give your answer in degrees, correct to two decimal places.

(b)

(i) Express $f(x) = \sqrt{3}\cos x - \sin x$ in the form $R\cos(x + \alpha)$, where $0 < \alpha < \frac{\pi}{2}$. 2

(ii) Sketch $y = f(x)$ for $0 \leq x \leq 2\pi$, showing clearly the location of all stationary points and the end-points. 2
Your sketch should be at least one-third of a page.

(iii) Hence, solve $\sqrt{3} \leq f(x) \leq 2$ for $0 \leq x \leq 2\pi$, expressing your solution using values in terms of π, where necessary. 1

Question 12 continues on the following page

Question 12 (continued)

(c) Use the principle of mathematical induction to show that for all integers $n \geq 1$, 3

$$1^2 + 3^2 + 5^2 + \cdots + (2n-1)^2 = \frac{4n^3 - n}{3}\ .$$

(d) Muckle Flugga Lighthouse is located on a small island and the lighthouse is located 3 km away from the main coastline. Assume the main coastline is a straight line. 3

Cooper is positioned at the point on the main coastline closest to the lighthouse.

If the light of the lighthouse revolves at 2 revolutions per minute, how fast is the beam of light moving along the coastline at a point 2 km away from Cooper?
Express your answer in terms of π .

Question 13 (15 marks)

Use a new sheet of paper

(a) (i) Sketch the function $y = f(x)$ where $f(x) = 9x - x^3$. 2

(ii) Hence, sketch $|y| = f(x-3)$. 2

(b) Use the substitution $u = \tan x$ to find the following integral: 2

$$\int \frac{\operatorname{cosec} x}{\cos x}\, dx$$

(c) Find the coefficient of x^3 upon expansion of the following: 3

$$\left(1 + \frac{2}{x^2}\right)^2 (2 - x)^{10}$$

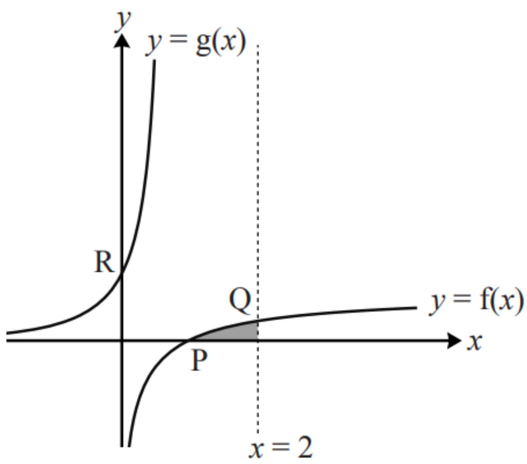
Question 13 continues on the following page

Question 13 (continued)

(d) The given diagram shows the curves $y = f(x)$ and $y = g(x)$ where:

$$f(x) = \ln\left(\frac{2x}{1+x}\right), \qquad x > 0$$

$$g(x) = \frac{e^x}{2-e^x}, \qquad x < \ln 2$$



The curve $y = f(x)$ crosses the x −axis at $P\ (1,0)$, and the line $x = 2$ at Q.

(i) Determine the y −coordinate of the point Q. 1

(ii) Find $f^{-1}(x)$, the inverse function of $f(x)$. 1

(iii) Show, using the substitution $u = 2 - e^x$, that: 2

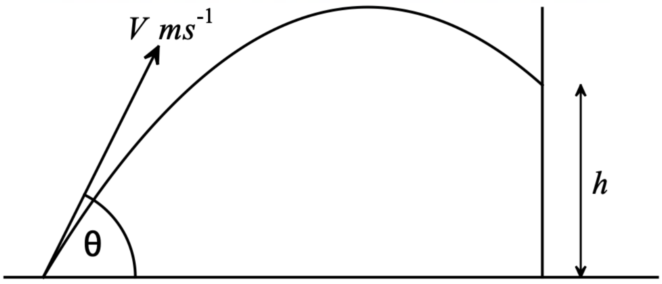
$$\int_0^{\ln\frac{4}{3}} g(x)dx = \ln\frac{3}{2}$$

(iv) Hence, find the area of the shaded region in the diagram, giving your answer in the form $\ln\frac{a}{b}$ where a and b are integers. 2

Question 14 (15marks)

Use a NEW sheet of paper

(a) A rocket is fired with a velocity V ms^{-1} at an angle θ to the horizontal from sea level and hits a cliff face as shown. Neglect air resistance and take $g = 10\ \text{m/s}^2$.



The position vector of the rocket, relative to the point from which it was fired is given by:

$$\underset{\sim}{s}(t) = 20t\underset{\sim}{i} + \left(20\sqrt{3}\,t - 5t^2\right)\underset{\sim}{j}$$

where t is the time in seconds after the rocket was fired.

(i) The base of the cliff is 120 m from the point at which the rocket was fired. **2**

Find the value of h, correct to the nearest metre.

(ii) Find the angle that the rocket makes **<u>with the cliff</u>** at the moment it strikes the cliff. **2**

Give your answer correct to the nearest minute.

(b) Consider the functions $y = -\frac{1}{2}\tan^{-1}(-2x)$ and $y = \cos^{-1}(2x)$.

(i) Sketch the function $y = \cos^{-1}(2x)$. **1**

(ii) Find the coordinates of any points of intersection and provide all values correct to 3 significant figures. **3**

Question 14 continues on the following page

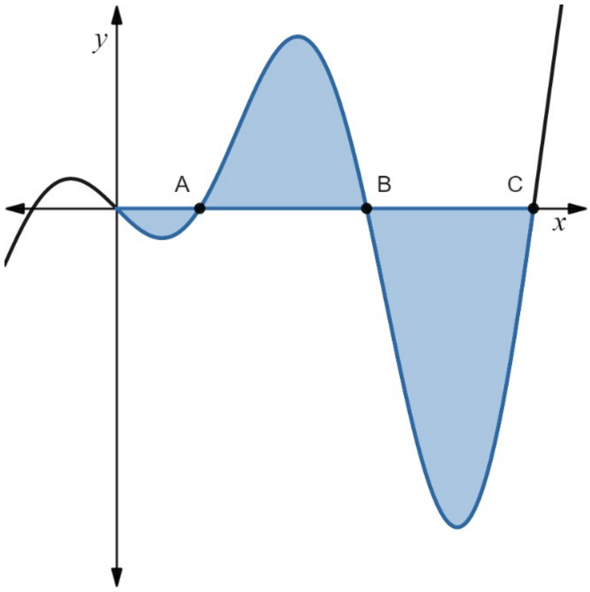
Question 14 (continued)

(c) The diagram below represents the graph of the function $y = \sin 5x - \sin 6x$.

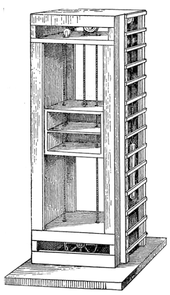
(i) Determine the exact coordinates of the point C . 2

(ii) Hence, calculate the volume of the solid formed when the shaded region is rotated by $360°$ about the x −axis. 3

Give your answer correct to 2 decimal places.



(d) The letters of the word DUMBWAITERS are arranged randomly.



Find the probability that:

(iii) the word <u>**WEIRD**</u> appears in the arrangement so that the letters are both in order and all together. 1

For example, the arrangement TUMB<u>**WEIRD**</u>AS.

(iv) the letters of the word <u>**WEIRD**</u> appear in the arrangement so that the letters are in order but not all together 1

For example, the arrangement U<u>**W**</u>TM<u>**E**</u>AB<u>**IRD**</u>S.

End of paper

SGHS X1 Trial HSC 2022: SOLUTIONS

Q01 *A*

For $f(x)$, the domain is $x \geq 0$ and the range is $y \geq -3$.

Hence, for $f^{-1}(x)$, the domain is $x \geq -3$ and the range is $y \geq 0$.

Q02 *A*

$$f'(x) = \frac{1}{1+\left(\frac{1}{x}\right)^2} \times -x^{-2}$$

$$= \frac{1}{1+\frac{1}{x^2}} \times -\frac{1}{x^2}$$

$$\therefore \quad f'(x) = -\frac{1}{x^2+1}$$

Q03 *C*

$$\frac{106}{5} = 21.2$$

$\therefore$ The minimum number of votes Terry can receive and still win is 22.

Q04 *B*

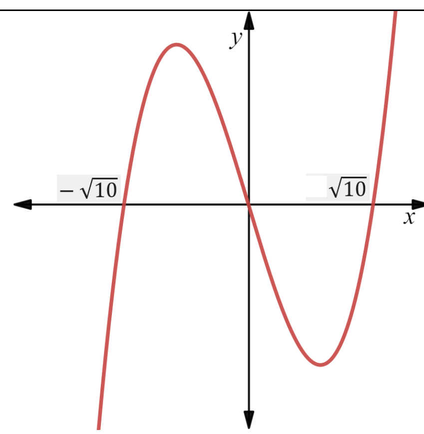
$x(x^2-10) < 0$

$x(x-\sqrt{10})(x+\sqrt{10}) < 0$

Using the graph on the right:

$x < -\sqrt{10}$ or $0 < x < \sqrt{10}$

i.e. $(-\infty, -\sqrt{10}) \cup (0, \sqrt{10})$



Q05 *C*

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= \frac{4t^3+4t}{3t^2+1}$$

when $t = 1, x = 2, y = 3$

$$m_T = \frac{4+4}{3+1}$$

$$m_T = 2$$

Eqn of tangent is $y - 3 = 2(x-2)$

i.e. $\quad y = 2x - 1$

Q06 $\quad A$

$$proj_{\underset{\sim}{w}}\underset{\sim}{F} = \frac{\underset{\sim}{F}.\underset{\sim}{w}}{\underset{\sim}{w}.\underset{\sim}{w}}\underset{\sim}{w}$$

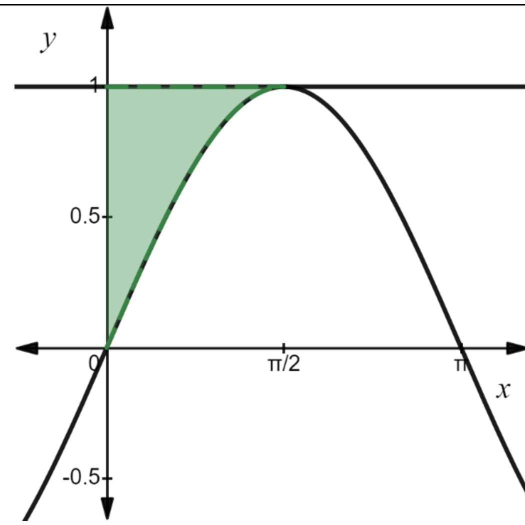
$$= \frac{a \times 1 + b \times 1}{1 \times 1 + 1 \times 1}\underset{\sim}{w}$$

$$\therefore \quad proj_{\underset{\sim}{w}}\underset{\sim}{F} = \frac{a+b}{2}\underset{\sim}{w}$$

Q07 $\quad D$

$$V = \pi \int_0^{\frac{\pi}{2}} 1^2 dx - \pi \int_0^{\frac{\pi}{2}} (\sin x)^2\, dx$$

$$\therefore \quad V = \pi \int_0^{\frac{\pi}{2}} (1 - \sin^2 x) dx$$



Q08 $\quad C$

No. of ways to order the three boys $= 3!$

$\_B\_B\_B\_$ $\quad$ No. of seating choices for the three girls $= 4 \times 3 \times 2$

$\therefore$ Total no of ways for seating $= 3! \times 24 = 144$

Q09 $\quad B$

$$\tan A - \tan B = m$$

$$\cot A - \cot B - 1 = m$$

$$\frac{1}{\tan A} - \frac{1}{\tan B} = m + 1$$

$$\frac{\tan B - \tan A}{\tan A \tan B} = m + 1$$

$$\frac{-m}{\tan A \tan B} = m + 1$$

$$\tan A \tan B = \frac{-m}{m+1}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$= \frac{m}{1 - \frac{m}{m+1}}$$

$$= \frac{m(m+1)}{m + 1 - m}$$

$$\therefore \quad \tan(A - B) = m(m+1)$$

Q10 $\quad D$

There will be 302 terms containing $\left((x)^{\frac{1}{2}}\right)^a \left((y)^{\frac{1}{5}}\right)^b$

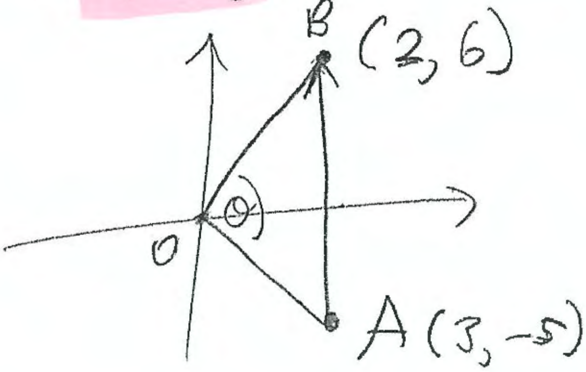
where $a + b = 301$ and $0 \le a, b \le 301$.

The rational terms occur when a is a multiple of 2 **and** b is a multiple of 5. Since a will be even and $a + b$ is odd, b must be odd .

Hence, there will be 30 rational terms, when $b = 5,15,25, \ldots ,295$.

2022 – Ext. 1 Trial

Q 11 a)



(i) $|\overrightarrow{OA}| = \sqrt{3^2+5^2}$

$= \sqrt{34}$ units

1

(ii) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -\overrightarrow{OA} + \overrightarrow{OB}$

$= -(3\underset{\sim}{i} - 5\underset{\sim}{j}) + 2\underset{\sim}{i} + 6\underset{\sim}{j}$

$= -\underset{\sim}{i} + 11\underset{\sim}{j}$

1

(iii) $OA = \sqrt{34}$ units

$OB = \sqrt{2^2+6^2}$

$= \sqrt{40}$

Let $\angle AOB = \theta$

$\underset{\sim}{a} \cdot \underset{\sim}{b} = |\underset{\sim}{a}||\underset{\sim}{b}| \cos\theta$

$RHS = \sqrt{34} \times \sqrt{40} \times \cos\theta$

$= \sqrt{1360}\ \cos\theta$

$LHS = \underset{\sim}{a} \cdot \underset{\sim}{b}$

$= x_1 x_2 + y_1 y_2$

$= 3 \times 2 + -5 \times 6$

$= 6 - 30$

$= -24$

$\cos\theta = \dfrac{-24}{\sqrt{1360}}$

$\theta \doteqdot 49^\circ$

$\angle AOB = 180^\circ - 49^\circ$

$\doteqdot 131^\circ$

(to nearest deg.)

2

11(b) $\displaystyle\int_0^{\frac{1}{3}} \frac{1}{\sqrt{4-9x^2}}\,dx$

$$= \frac{1}{3}\int_0^{\frac{1}{3}} \frac{3}{\sqrt{2^2-(3x)^2}}\,dx$$

$$= \frac{1}{3}\left[\sin^{-1}\left(\frac{3x}{2}\right)\right]_0^{\frac{1}{3}}$$

$$= \frac{1}{3}\left(\sin^{-1}\left(\frac{1}{2}\right) - \sin^{-1}(0)\right)$$

$$= \frac{1}{3}\left(\frac{\pi}{6} - 0\right)$$

$$= \frac{\pi}{18}$$

/2

11c) $p(x) = x^3 + ax^2 - 6ax + b$

$p(2) = 0$

$p(1) = 2$

$8 + 4a - 12a + b = 0$

$8a - b = 8$ ...... (1)

$1 + a - 6a + b = 2$

$5a - b = -1$ ...... (2)

(1) − (2): $3a = 9$

$a = 3$

Sub a in (2):

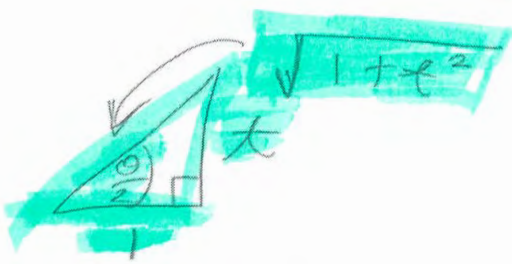
$15 - b = -1$

$b = 16$

$\therefore a = 3, \ b = 16$

/3

11 d) (i) $t = \tan\left(\frac{\theta}{2}\right)$



1

$$\sin\left(\frac{\theta}{2}\right) = \frac{t}{\sqrt{1+t^2}}$$

(ii) $$\cos\theta = \frac{1-t^2}{1+t^2}$$

$$\cos\theta - \sin\left(\frac{\theta}{2}\right) = 0$$

$$\frac{1-t^2}{1+t^2} = \frac{t}{\sqrt{1+t^2}}$$

$$1-t^2 = \frac{t(1+t^2)}{(1+t^2)^{\frac{1}{2}}}$$

$$1-t^2 = t(1+t^2)^{\frac{1}{2}}$$

$$(1-t^2)^2 = t^2(1+t^2)$$

$$1-2t^2+\cancel{t^4} = t^2+\cancel{t^4}$$

$$3t^2 = 1$$

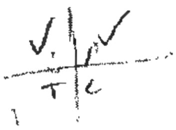
$$t^2 = \frac{1}{3}$$

$$t = \pm\frac{1}{\sqrt{3}}$$

$0 \leq \theta \leq 2\pi$

$0 \leq \frac{\theta}{2} \leq \pi$

$$\tan\left(\frac{\theta}{2}\right) = \pm\frac{1}{\sqrt{3}}$$



$$\frac{\theta}{2} = \frac{\pi}{6} \qquad \pi - \frac{\pi}{6}$$

$$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

3

11e) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -\underset{\sim}{a} + \underset{\sim}{b}$

$\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BD}$

$= (\underset{\sim}{b} - \underset{\sim}{a}) + \frac{3}{2} \times -\underset{\sim}{a}$

$= \underset{\sim}{b} - \frac{5}{2}\underset{\sim}{a}$

/2

$\boxed{\text{OR}}$

$\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{BD}$

$= \underset{\sim}{b} + -\frac{3}{2}\underset{\sim}{a}$

$\overrightarrow{AD} = \overrightarrow{AO} + \overrightarrow{OD}$

$= -\underset{\sim}{a} + (\underset{\sim}{b} - \frac{3}{2}\underset{\sim}{a})$

$= \underset{\sim}{b} - \frac{5}{2}\underset{\sim}{a}$

Question 12

(a)

(i) $T = A + Pe^{kt}$ (1)

$\frac{dT}{dt} = kPe^{kt}$

$= k(T-A)$, since $Pe^{kt} = T - A$ from (1)

$\therefore$ (1) satisfies the DE. ✓

Comment : Generally well done. There were a handful of students that were not successful with this question - it is recommended to learn the above setting out.

(ii) $A = 25$

$T = 25 + Pe^{kt}$

$t=0$, $T=2$

$2 = 25 + Pe^{0}$

$\therefore -23 = P$ ✓

$T = 25 - 23e^{kt}$

$t = 15$, $T = 10$

$10 = 25 - 23e^{15k}$

$-15 = -23e^{15k}$

$e^{15k} = \frac{15}{23}$

$15k = \ln\frac{15}{23}$

$\therefore k = \frac{1}{15}\ln\frac{15}{23}$ ✓

When $t = 30$,

$T = 25 - 23e^{30k}$

$= 15.22°C$ (2 dp) ✓

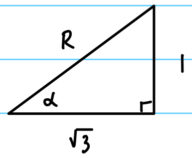
(b)

(i) $\sqrt{3}\cos x - \sin x = R\cos(x+\alpha)$

$= R\cos x\cos\alpha - R\sin x\sin\alpha$

$R\cos\alpha = \sqrt{3}$

$R\sin\alpha = 1$



$R = \sqrt{(\sqrt{3})^2 + 1^2}$

$= 2$

$\tan\alpha = \frac{1}{\sqrt{3}}$

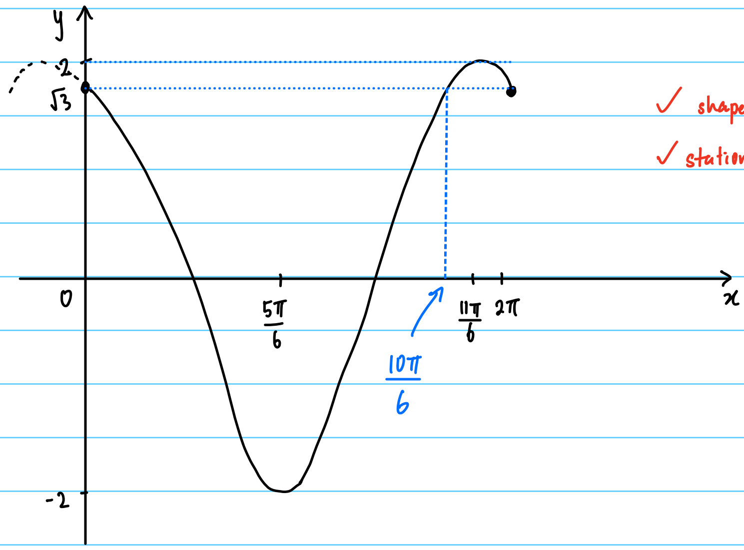
$\alpha = \frac{\pi}{6}$

✓✓

$\therefore f(x) = 2\cos\left(x + \frac{\pi}{6}\right)$

Comment: Generally well done. Proper derivation of the values of R and α are needed to obtain full marks.

(ii)



✓ shape and position

✓ stationary and endpoints

Comment: Students who recognised that $f(x) = 2\cos\left(x+\frac{\pi}{6}\right)$ is simply a shift of $y = 2\cos x$ by $\frac{\pi}{6}$ units to the left were generally the most successful.

Many students inefficiently derived the coordinates of the stationary points by solving trigonometric equations and/or by differentiating $f(x)$.

(iii) $x = 0$, $\frac{5\pi}{3} \le x \le 2\pi$ ✓

Comment: Many students left out $x = 0$ as part of their solution.

(c) Prove true for $n=1$

$LHS = 1^2 = 1$

$RHS = \dfrac{4(1)^3 - 1}{3} = 1$

$\therefore LHS = RHS$

$\therefore$ true for $n=1$ ✓

Assume true for $n = k$

ie. assume

$$1^2 + 3^2 + 5^2 + \dots + (2k-1)^2 = \frac{4k^3 - k}{3}$$

Prove true for $n = k+1$

ie. RTP

$$1^2 + 3^2 + 5^2 + \dots + (2k+1)^2 = \frac{4(k+1)^3 - (k+1)}{3}$$

$LHS = \dfrac{4k^3 - k}{3} + (2k+1)^2$ (by assumption)

$= \dfrac{4k^3 - k + 3(4k^2 + 4k + 1)}{3}$ ✓

$= \dfrac{4k^3 + 12k^2 + 12k + 4 - k - 1}{3}$

$= \dfrac{4(k^3 + 3k^2 + 3k + 1) - (k+1)}{3}$ ✓

$= \dfrac{4(k+1)^3 - (k+1)}{3}$

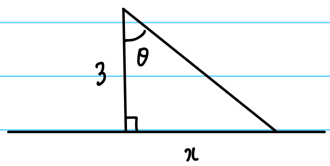
$= RHS$

$\therefore$ true for $n = k+1$

$\therefore$ true by induction for integers $n \geq 1$.

Comment: Students who knew that $(k+1)^3 = k^3 + 3k^2 + 3k + 1$ were generally the most successful.

(d)



(θ, x, 3 must ALL be labelled)

$$\frac{d\theta}{dt} = 2 \text{ revs/min}$$

$$= 4\pi \text{ rads/min}$$

$$\frac{dx}{dt} = \frac{d\theta}{dt} \times \frac{dx}{d\theta}$$

$$= 4\pi \times \frac{9+x^2}{3}$$

$$\tan\theta = \frac{x}{3}$$

$$\theta = \tan^{-1}\frac{x}{3}$$

$$\frac{d\theta}{dx} = \frac{\frac{1}{3}}{1+\frac{x^2}{9}} = \frac{3}{9+x^2}$$

When $x = 2$,

$$\frac{dx}{dt} = 4\pi \times \frac{9+2^2}{3}$$

$$= \frac{52\pi}{3} \text{ km/min}$$

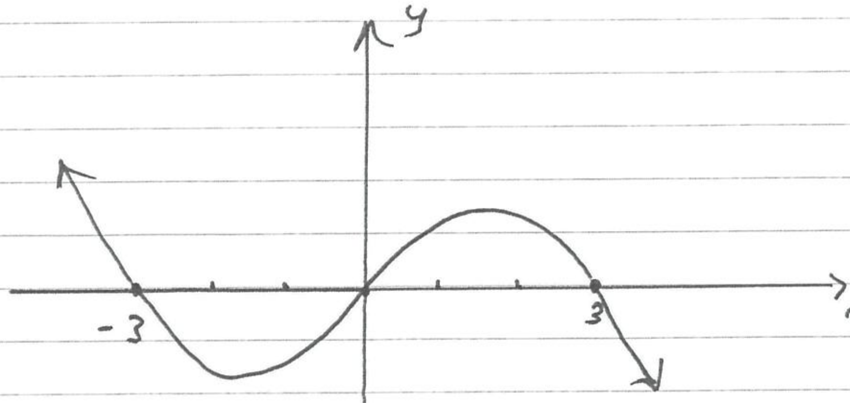
Comment: Only a handful of students were successful with this question. Students who able to correctly depict the situation with a labelled diagram were generally the most successful.

Q13 Ext 1 2022 Trial

a) i) $y = 9x - x^3$

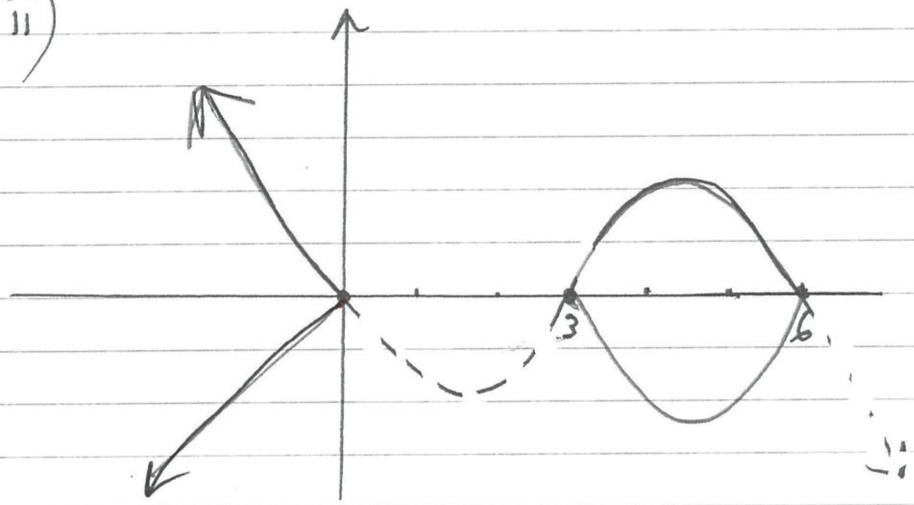
$= x(9 - x^2)$

$= x(3 - x)(3 + x)$



✓✓

ii)



✓✓

This question was done poorly. Many students didn't know how to do it.

b) $u = \tan x$

$\frac{du}{dx} = \sec^2 x$

$du = \sec^2 x \, dx$

$$\int \frac{1}{\sin x \cos x} \times \frac{\cos x}{\cos x} \, dx$$

$$= \int \frac{\cos x}{\sin x \cos^2 x} \, dx = \int \frac{\cos x \sec^2 x}{\sin x} \, dx$$

$= \int \cot x \sec^2 x \, dx$

$= \int \frac{1}{\tan x} \sec^2 x \, dx$

$= \int \frac{1}{u} \, du$

$= \ln|u| + c$

$= \ln|\tan x| + C$

c) $\left(1 + \frac{4}{x^2} + \frac{4}{x^4}\right)\left({}^{10}C_7\, 2^7(-x)^3 + {}^{10}C_5\, 2^5(-x)^5 + {}^{10}C_3\, 2^3(-x)^7\right)$

$= {}^{10}C_7\, 2^7(-x)^3 + \frac{4}{x^2}\left({}^{10}C_5\, 2^5(-x)^5\right) + \frac{4}{x^4}\left({}^{10}C_3\, 2^3(-x)^7\right)$

$= -15360x^3 - 32256x^3 - 3840x^3$

$= -51456\, x^3$

coefficient -51456

This question was very poorly done. students didn't have clear setting out.

d) i) $y = \ln\left(\frac{2x}{1+x}\right)$

$x = 2$

$y = \ln\left(\frac{4}{3}\right)$

$P\left(2, \ln\frac{4}{3}\right)$

ii) $y = \ln\left(\frac{2x}{1+x}\right)$

$x = \ln\left(\frac{2y}{1+y}\right)$

$e^x = \frac{2y}{1+y}$

$e^x + ye^x = 2y$

$e^x = 2y - ye^x$

$e^x = y(2 - e^x)$

$y = \frac{e^x}{2-e^x}$

iii) $u = 2 - e^x$

$\frac{du}{dx} = -e^x$

$\int_0^{\ln\frac{4}{3}} \frac{e^x}{2-e^x}\,dx$

$-\int_0^{\ln\frac{4}{3}} \frac{-e^x}{2-e^x}\,dx$

$= -\int \frac{du}{u}$

$u = 2 - e^x$

$x = 0 \rightarrow u = 2 - 1$

$u = 1$

$x = \ln\frac{4}{3} \rightarrow u = 2 - e^{\ln\frac{4}{3}}$

$= 2 - \frac{4}{3}$

$= \frac{2}{3}$

$-\int_1^{\frac{2}{3}} \frac{du}{u}$

$-\left[\ln|u|\right]_1^{\frac{2}{3}}$

$= -\left[\ln\left(\frac{2}{3}\right) - \ln(1)\right]$

$= -\ln\left(\frac{2}{3}\right)$

$= \ln\left(\frac{2}{3}\right)^{-1}$

$= \ln\frac{3}{2}$

iv)

$$A = \left(\ln \frac{4}{3}\right) \times 2 - \int_0^{\ln \frac{4}{3}} \frac{e^y}{2 - e^y}\, dy$$

$$= 2\ln \frac{4}{3} - \left[\ln \frac{3}{2}\right]$$

$$= \ln \frac{16}{9} - \ln \frac{3}{2}$$

$$= \ln \left(\frac{\frac{16}{9}}{\frac{3}{2}}\right)$$

$$= \ln \left(\frac{32}{27}\right) \; u^2$$

This question was hard for many students

Q 14

a) i) $s(t) = 20t\,\underset{\sim}{i} + (20\sqrt{3}t - 5t^2)\,\underset{\sim}{j}$

$x = 20t$ (1)

$y = 20\sqrt{3}\,t - 5t^2$ (2)

from (1) $t = \frac{x}{20}$ subs into (2)

$y = 20\sqrt{3} \times \frac{x}{20} - \frac{5 \times x^2}{400}$ ✓

$y = \sqrt{3}\,x - \frac{x^2}{80}$

When $x = 120$

$y = \sqrt{3} \times 120 - \frac{120^2}{80} = \boxed{28\text{ m}}$ ✓

ii) when it hits the ~~cliff~~ $x = 120$m

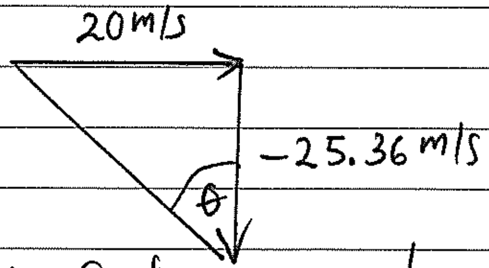
$t = \frac{120}{20} = 6$ sec

$x = 20t$ $\qquad$ $y = 20\sqrt{3}t - 5t^2$

$\dot{x} = 20$ m/s $\qquad$ $\dot{y} = 20\sqrt{3} - 10t$ ✓

At $t = 6 \therefore \dot{y} = 20\sqrt{3} - 10 \times 6$

$\dot{y} = -25.36$ m/s



$\tan\theta = \left|\frac{20}{-25.36}\right|$

$\theta = \boxed{38^\circ 16'}$ ✓

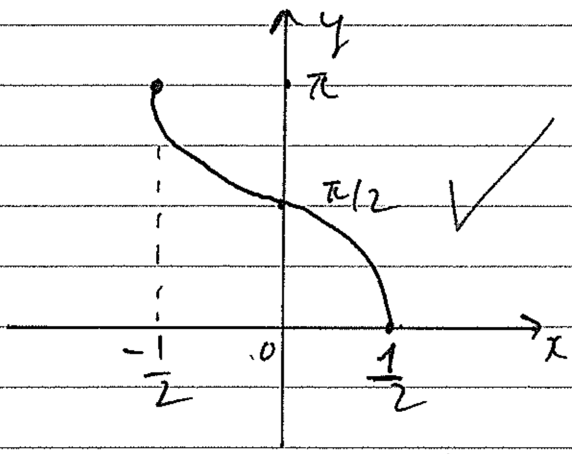
Note: To find the angle, the horizontal Vel and vertical Vel must be calculated (Not horizontal and vertical distances)

Q14

b) i) $y = \cos^{-1} 2x$

D: $-1 \le 2x \le 1$

R: $0 \le y \le \pi$



ii) solve $y = -\frac{1}{2}\tan^{-1}(-2x)$, $y = \cos^{-1}(2x)$

$$\cos^{-1}(2x) = -\frac{1}{2}\tan^{-1}(-2x)$$

$$\cos^{-1}(2x) = \frac{1}{2}\tan^{-1}(2x)$$

$$2\cos^{-1}(2x) = \tan^{-1}(2x)$$

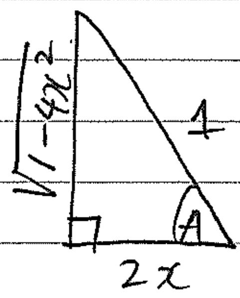
Insert tan for both sides

$$\tan 2\left[\cos^{-1}(2x)\right] = \tan\left(\tan^{-1}(2x)\right) \quad (*)$$

$\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$ let $A = \cos^{-1} 2x$

$\cos A = 2x$

$\tan A = \dfrac{\sqrt{1-4x^2}}{2x}$



from (*)

$$\frac{\dfrac{2\times\sqrt{1-4x^2}}{2x}}{1-\dfrac{(1-4x^2)}{4x^2}} = 2x$$

$$\frac{4x\sqrt{1-4x^2}}{8x^2-1} = 2x$$

$$2\sqrt{1-4x^2} = 8x^2 - 1$$

$$4(1-4x^2) = 64x^4 - 16x^2 + 1$$

$$4 - 16x^2 = 64x^4 - 16x^2 + 1$$

$$64x^4 = 3$$

$$x = \pm\sqrt[4]{\frac{3}{64}}$$

$$x = 0.465 \longrightarrow y = 0.375$$

At $x = -0.465$ (two fns do not intersect)

Point of intersection : $(0.465, 0.375)$ ✓

c)

i) ~~$y = 5x$~~

$y = \sin 5x - \sin 6x$

At C, $y = 0$

$$\sin 5x - \sin 6x = 0$$

Use : $\sin(A-B) - \sin(A+B) = -2\cos A.\sin B$

$$\sin 5x - \sin 6x = -2\cos\frac{11x}{2} . \sin\frac{x}{2}$$ ✓

OR $$-2\cos\frac{11x}{2} . \sin\frac{x}{2} = 0$$

$$\left[\begin{array}{l} \cos\frac{11x}{2} = 0 \\ \sin\frac{x}{2} = 0 \end{array}\right. \quad \therefore \left[\begin{array}{l} x = \frac{\pi}{11}, \frac{3\pi}{11}, \frac{5\pi}{11} \ldots \\ x = 0, 2\pi, 4\pi \ldots \end{array}\right.$$

Arrange the sol^n in order

$$x = 0, \frac{\pi}{11}, \frac{3\pi}{11}, \frac{5\pi}{11} \ldots$$

At point C, the 4th solution

$$\therefore x = \frac{5\pi}{11} \text{ at } C$$

$$C\left(\frac{5\pi}{11}, 0\right)$$

ii) $$V = \pi \int_0^{\frac{5\pi}{11}} (\sin 5x - \sin 6x)^2 \, dx$$

$$V = \pi \int_0^{5\pi/11} (\sin^2 5x + \sin^2 6x - 2\sin 5x . \sin 6x) \, dx$$

Note: $\sin^2 A = \frac{1}{2}(1 - \cos 2A)$

$$\sin^2 5x = \frac{1}{2}(1 - \cos 10x)$$

$$\sin^2 6x = \frac{1}{2}(1 - \cos 12x)$$

$$-2\sin 5x . \sin 6x = -2\left[\frac{1}{2}(\cos(-x) - \cos 11x)\right]$$

$$V = \pi \int_0^{5\pi/11} \left(\frac{1}{2} - \frac{\cos(10x)}{2} + \frac{1}{2} - \frac{\cos(12x)}{2} - \cos x + \cos 11x\right) dx$$

$$V = \pi \left[x - \frac{\sin(10x)}{20} - \frac{\sin(12x)}{24} - \sin x + \frac{\sin 11x}{11}\right]_0^{5\pi/11}$$

Q14

$$V = \pi\left[\frac{5\pi}{11} - \frac{1}{20}\sin\left(\frac{50\pi}{11}\right) - \frac{1}{24}\sin\left(\frac{60\pi}{11}\right) - \sin\frac{5\pi}{11} + \frac{1}{11}\sin 5\pi\right]$$

$V = 1.35$ cubic units

d) i) $\square\,\square$ W E I R D $\square\,\square\,\square\,\square$

$$= \frac{7!}{11!} = \frac{1}{7920}$$

ii)

W E I R D

$\square$

1st Letter

$= 6 \times 7 \times 8 \times 9 \times 10 \times 11$

1st Letter, 2nd Letter, 3rd Letter

P(WEIRD IN ORDER But Not All Together)

= All possible of WEIRD in order − WEIRD in order and together.

$$= \frac{6 \times 7 \times 8 \times 9 \times 10 \times 11}{11!} - \frac{7!}{11!}$$

$$= \frac{13}{1584}$$